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Similarity of signal processing effect between Hankel matrix-based SVD and wavelet transform and its mechanism analysis

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A R T I C L E I N F O

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It is pointed out that signal processing effect of singular value decomposition (SVD) is very similar to that of wavelet transform when Hankel matrix is used. It is proved that a signal can be decomposed into the linear sum of a series of component signals by Hankel matrix-based SVD, and essentially what the component signals reflect are projections of original signal on the orthonormal bases of m-dimensional and n-dimensional vector spaces. The similarity mechanism of signal processing between SVD and wavelet transform is analyzed from the angle of basis of vector space and characteristic of Hankel matrix. The orthogonality of the component signals got by SVD and wavelet transform is also studied. It is discovered that singularity of signal can also be detected by Hankel matrix-based SVD, and compared with wavelet transform, there are two characteristics in SVD for singularity detection, one is that the order of vanishing moment of SVD component signals is increased progressively and the one of the nth SVD component signal is '$n-1$', so singular points with different Lip index can all be detected, the other is that the width of impulse indicating the position of singularity will always keep the same throughout all SVD components and this width is determined by the column number of Hankel matrix.

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1. Introduction

The definition of singular value decomposition (SVD) is: for a matrix $\boldsymbol{A}\in\mathbf{R}^{m\times n}$, two orthogonal matrices $\boldsymbol{U}=[\boldsymbol{u}_1, \boldsymbol{u}_2, \ldots, \boldsymbol{u}_m]\in\mathbf{R}^{m\times m}$ and $\boldsymbol{V}=[\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n]\in\mathbf{R}^{n\times n}$ are surely existed to meet the following equation [1]

$$\boldsymbol{A}=\boldsymbol{USV}^T \tag{1}$$

where $\boldsymbol{S}=[diag(\sigma_1, \sigma_2, \ldots, \sigma_q), \boldsymbol{O}]$ or its transposition, which is decided by $m<n$ or $m>n$, $\boldsymbol{S}\in\mathbf{R}^{m\times n}$, while $\boldsymbol{O}$ is zero matrix, $q=\min(m, n)$, and $\sigma_1\geqslant\sigma_2\geqslant\cdots\geqslant\sigma_q>0$. These σ_i $(i=1, 2, \ldots, q)$ are called the singular values of matrix $\boldsymbol{A}$.

SVD and wavelet transform are two different theories that can never be mentioned in the same breath and their mathematical foundations are completely different from each other, but we discovered that if suitable matrix $\boldsymbol{A}$ is adopted, SVD will show the very surprisingly similar effect of signal processing to that of wavelet transform. Many kinds of matrices can be created by one-dimensional signal, such as Toeplitz matrix, cycle matrix and Hankel matrix, or matrix can be created by continuous interception of signal. The difference is the creation way of matrix, the difference will be the effect of signal processing of SVD [2]. To our surprise, SVD can achieve the very similar effect of signal processing to wavelet transform when $\boldsymbol{A}$ is Hankel matrix, such as that the original signal can be decomposed into the combination of a series of detail

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signals and approximation signal by both methods, and the singular points in signal can be detected by both methods. Furthermore, in some aspects, the decomposition results of SVD have the special advantage of those of the wavelet transform. Why the signal processing effect of these two completely different theories are so similar to each other? This question is worth studying.

To explain this phenomenon reasonably, the essence of signal decomposition of these two methods should be ascertained. As is well known, in fact the essence of wavelet transform is to use two high and low pass filters to decompose a signal into a series of component signals locating in different frequency bands, while as for SVD, there is no such direct explanation. Most references on the signal processing of SVD always place their emphasis on practicality and SVD is only used as a tool to solve certain specific signal processing problem, such as feature extraction [3–6], data compression [7], noise reduction [8–11], speech coding [12] and so on, while the study on the essence of signal decomposition of SVD is very few, in this paper this essence has been ascertained, and on this basis the similarity mechanism of signal processing between SVD and wavelet transform is explained from the angle of basis of vector space. Besides, the orthogonality of component signals, got by these two methods, is also studied and compared theoretically.

On the basis of theory analysis, the practically similar effect of signal processing of these two methods are demonstrated by several examples, and it is pointed out that for structure characteristic of Hankel matrix in itself, the first SVD component signal can correspond to the approximation signal of the last scale in wavelet transform, while the other SVD component signals can correspond to the detail signals in the wavelet transform.

On the other hand, some differences between these two methods are also studied, such as that there is no phase shift in the decomposition results of SVD, while phase lag is existed in those of wavelet transform. And as to the singularity detection, for that the order of vanishing moment of each kind of wavelet is fixed, once wavelet is selected, the singular index that can be detected by wavelet transform will also be fixed, and, moreover, the width of impulse indicating the position of singular point will become broad and broad with the increase of scale. While SVD is different from this, it is discovered that the order of vanishing moment of SVD components is increased progressively and the one of the nth SVD component signal is '$n-1$', so singular points with different singular index can also be detected by different SVD components, besides, the width of impulse indicating the position of singular point will always keep the same throughout all SVD components and it is found that this width is determined by the column number of Hankel matrix, so in these aspects, SVD has the advantage of wavelet transform.

2. Essence of signal decomposition of Hankel matrix-based SVD

For a discrete signal $\boldsymbol{X} = [x(1), x(2), \ldots, x(N)]$, Hankel matrix can be created by this signal as follows:

$$\boldsymbol{A} = \begin{bmatrix} x(1) & x(2) & \cdots & x(n) \\ x(2) & x(3) & \cdots & x(n+1) \\ \vdots & \vdots & \vdots & \vdots \\ x(N-n+1) & x(N-n+2) & \cdots & x(N) \end{bmatrix}$$

where $1<n<N$, let $m = N-n+1$, then $\boldsymbol{A} \in \mathbf{R}^{m\times n}$.

In order to realize the isolation of signal using SVD, the Eq. (1) should be converted to the form of column vectors $\boldsymbol{u}_i$ and $\boldsymbol{v}_i$

$$\boldsymbol{A} = \sigma_1 \boldsymbol{u}_1 \boldsymbol{v}_1^T + \sigma_2 \boldsymbol{u}_2 \boldsymbol{v}_2^T + \cdots + \sigma_q \boldsymbol{u}_q \boldsymbol{v}_q^T \tag{2}$$

where $\boldsymbol{u}_i \in \mathbf{R}^{m\times 1}$, $\boldsymbol{v}_i \in \mathbf{R}^{n\times 1}$, $i = 1, 2, \ldots, q$, $q = \min(m, n)$. According to the SVD theory, the vectors $\boldsymbol{u}_i$ are orthonormal to one another and they form the orthonormal bases of m-dimensional space; the vectors $\boldsymbol{v}_i$ are also orthonormal to one another and they form the orthonormal bases of n-dimensional space [1].

Let $\boldsymbol{A}_i = \sigma_i \boldsymbol{u}_i \boldsymbol{v}_i^T$, then $\boldsymbol{A}_i \in \mathbf{R}^{m\times n}$ also. Supposing that $\boldsymbol{P}_{i,1}$ is the first row vector of $\boldsymbol{A}_i$, and $\boldsymbol{H}_{i,n}$ is a column vector in the last column of $\boldsymbol{A}_i$, which is shown in Fig. 1, according to the creation principle of Hankel matrix, if $\boldsymbol{P}_{i,1}$ and the transposition of $\boldsymbol{H}_{i,n}$ are linked together, then a SVD component signal $\boldsymbol{P}_i$ can be obtained, which can be expressed as the vector form

$$\boldsymbol{P}_i = (\boldsymbol{P}_{i,1}, \boldsymbol{H}_{i,n}^T). \quad \boldsymbol{P}_{i,1} \in \boldsymbol{R}^{1\times n}, \quad \boldsymbol{H}_{i,n} \in \boldsymbol{R}^{(m-1)\times 1} \tag{3}$$

$$\boldsymbol{A}_i = \begin{bmatrix} x_i(1) & x_i(2) & \cdots & x_i(n) \\ x_i(2) & x_i(3) & \cdots & x_i(n+1) \\ \vdots & \vdots & \vdots & \vdots \\ x_i(N-n+1) & x_i(N-n+2) & \cdots & x_i(N) \end{bmatrix} \begin{matrix} P_{i,1} \\ \\ \\ \\ \end{matrix}$$

$$H_{i,n}$$

Fig. 1. The forming principle of component signal $\boldsymbol{P}_i$ when Hankel matrix is used.

All the component signals formed by $\boldsymbol{A}_i$ ($i = 1, 2, \ldots, q$) make up one kind of decomposition for the original signal $\boldsymbol{X}$. However, what on earth do these component signals reflect in essence? To ascertain this question, firstly we might as well divide the component signal $\boldsymbol{P}_i$ into two segments, as is illustrated in Fig. 2, in which the front segment is $\boldsymbol{P}_{i,1}$, which is the first row vector of $\boldsymbol{A}_i$, while the back segment is the transposition of $\boldsymbol{L}_{i,n}$, where $\boldsymbol{L}_{i,n}$ is the last column vector of $\boldsymbol{A}_i$.

Supposing that Hankel matrix $\boldsymbol{A}$ created by the original signal is expressed by row vectors $\boldsymbol{X}_1, \boldsymbol{X}_2, \ldots, \boldsymbol{X}_m$, $\boldsymbol{X}_m \in \mathbf{R}^{1\times n}$. As is known from the creation of $\boldsymbol{A}$, the first vector $\boldsymbol{X}_1$ is the front segment of original signal and its projective coefficient on the basis vector $\boldsymbol{v}_i$ of n-dimensional can be computed from Eq. (1) as follows:

$$\boldsymbol{X}_1\boldsymbol{v}_i = \sigma_i u_{i1} \tag{4}$$

where u_{i1} is the first coordinates of basis vector $\boldsymbol{u}_i$.

While for $\boldsymbol{P}_{i,1}$, which is the front segment of component signal $\boldsymbol{P}_i$ and also the first row vector of matrix $\boldsymbol{A}_i$, it can be computed according to the definition of $\boldsymbol{A}_i$ as below:

$$\boldsymbol{P}_{i,1} = \sigma_i u_{i1}\boldsymbol{v}_i^T \tag{5}$$

One can easily see that $\boldsymbol{P}_{i,1}$ is the product of basis vector $\boldsymbol{v}_i$ and projective coefficient of $\boldsymbol{X}_1$ on this basis vector, in which projective coefficient $\sigma_i u_{i1}$ determines the magnitude of $\boldsymbol{P}_{i,1}$, and vector $\boldsymbol{v}_i$ determines the direction of $\boldsymbol{P}_{i,1}$. While $\boldsymbol{X}_1$ is the front segment of original signal, so obviously $\boldsymbol{P}_{i,1}$ is actually the projection of the front segment of original signal on the vector $\boldsymbol{v}_i$, which is the ith basis vector of n-dimensional space, and this relationship is illustrated in Fig. 3(a).

Supposing that Hankel matrix $\boldsymbol{A}$ created by original signal is expressed by the column vectors $\boldsymbol{Y}_1, \boldsymbol{Y}_2, \ldots, \boldsymbol{Y}_n$, $\boldsymbol{Y}_n \in \mathbf{R}^{m\times 1}$, as is known from the creation of $\boldsymbol{A}$, the last column vector $\boldsymbol{Y}_n$ is the back segment of original signal and its projective coefficient on the basis vector $\boldsymbol{u}_i$ of m-dimensional can also be computed from Eq. (1) as follows:

$$\boldsymbol{Y}_n^T\boldsymbol{u}_i = \sigma_i \boldsymbol{v}_{in} \tag{6}$$

where v_{in} is the nth coordinates of basis vector $\boldsymbol{v}_i$.

While for $\boldsymbol{L}_{i,n}$, which is the back segment of component signal $\boldsymbol{P}_i$ and also the last column vector of matrix $\boldsymbol{A}_i$, it can be computed according to the definition of $\boldsymbol{A}_i$ as below:

$$\boldsymbol{L}_{i,n}^T = \sigma_i v_{in}\boldsymbol{u}_i^T \tag{7}$$

It can be seen that $L_{i,n}{}^{\mathrm{T}}$ is the product of basis vector $\boldsymbol{u}_i$ and projective coefficient of $\boldsymbol{Y}_n$ on this basis vector, in which projective coefficient $\sigma_i v_{in}$ determines the magnitude of $L_{i,n}{}^{\mathrm{T}}$, and vector $\boldsymbol{u}_i$ determines the direction of $L_{i,n}{}^{\mathrm{T}}$. While $\boldsymbol{Y}_n$ is the back segment of original signal, so $L_{i,n}{}^{\mathrm{T}}$ is actually the projection of back segment of original signal on the vector $\boldsymbol{u}_i$, which is the ith basis vector of m-dimensional space, and this relationship is illustrated in Fig. 3(b).

So we can see from the above analysis that the essence of signal decomposition of Hankel matrix-based SVD is in fact to decompose the signal into the two spaces, i.e. m-dimensional and n-dimensional spaces, and what component signals reflect are actually the projections of original signal on the orthonormal bases of these two spaces, i.e. the similarity extent between original signal and these two orthonormal bases.

Let us further analyze the characteristic of this kind of decomposition. Supposing that $\boldsymbol{A}_i$ is expressed by row vectors $\boldsymbol{P}_{i,1}, \boldsymbol{P}_{i,2}, \ldots, \boldsymbol{P}_{i,m}$, $\boldsymbol{P}_{i,m} \in \mathbf{R}^{1\times n}$, while $\boldsymbol{A}$ is expressed by row vectors $\boldsymbol{X}_1, \boldsymbol{X}_2, \ldots, \boldsymbol{X}_m$, $\boldsymbol{X}_m \in \mathbf{R}^{1\times n}$. According to Eq. (2), it is obvious that

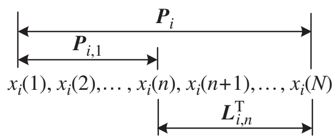

Fig. 2. The front and back segment of component signal $\boldsymbol{P}_i$.

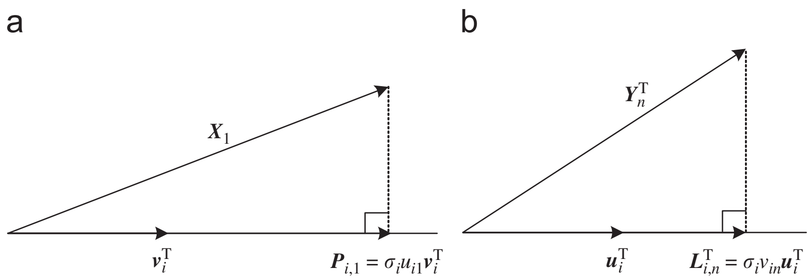

Fig. 3. The essence of component signal $\boldsymbol{P}_i$. (a) Essence of the front segment of $\boldsymbol{P}_i$ and (b) essence of the back segment of $\boldsymbol{P}_i$.

each row vector of $\boldsymbol{A}$ equals the sum of corresponding row vectors in all $\boldsymbol{A}_i$ ($i = 1, 2, \ldots, q$), thus we have

$$\boldsymbol{X}_1 = \boldsymbol{P}_{1,1} + \boldsymbol{P}_{2,1} + \cdots + \boldsymbol{P}_{q,1} \tag{8}$$

As to the column vector $\boldsymbol{H}_{i,n}$ in $\boldsymbol{A}_i$, $\boldsymbol{H}_{i,n} \in \mathbf{R}^{(m-1)\times 1}$, assume that the corresponding column vector in $\boldsymbol{A}$ is $\boldsymbol{I}_n$, $\boldsymbol{I}_n \in \mathbf{R}^{(m-1)\times 1}$, it is obvious from Eq. (2) that $\boldsymbol{I}_n$ also equals the sum of the corresponding column vectors $\boldsymbol{H}_{i,n}$ in all $\boldsymbol{A}_i$ ($i = 1, 2, \ldots, q$), and their transposition also satisfy this relation, i.e.

$$\boldsymbol{I}_n^T = \boldsymbol{H}_{1,n}^T + \boldsymbol{H}_{2,n}^T + \cdots + \boldsymbol{H}_{q,n}^T \tag{9}$$

While according to the creation principle of Hankel matrix, original signal $\boldsymbol{X}$ can be expressed as vector form $\boldsymbol{X} = (\boldsymbol{X}_1, \boldsymbol{I}_n^{\mathrm{T}})$, and component signal $\boldsymbol{P}_i$ can also be expressed as vector form $\boldsymbol{P}_i = (\boldsymbol{P}_{i,1}, \boldsymbol{H}_{i,n}^{\mathrm{T}})$, then the sum of all these component signals are

$$\boldsymbol{P}_1 + \boldsymbol{P}_2 + \cdots + \boldsymbol{P}_q = (\boldsymbol{P}_{1,1} + \boldsymbol{P}_{2,1} + \cdots + \boldsymbol{P}_{q,1}, \boldsymbol{H}_{1,n}^T + \boldsymbol{H}_{2,n}^T + \cdots + \boldsymbol{H}_{q,n}^T)$$

While according to Eqs. (8) and (9), the right part of above formula can be written as $(\boldsymbol{X}_1, \boldsymbol{I}_n^{\mathrm{T}})$, so

$$\boldsymbol{P}_1 + \boldsymbol{P}_2 + \cdots + \boldsymbol{P}_q = \boldsymbol{X} \tag{10}$$

From Eq. (10) it can be seen that when Hankel matrix is used, the component signals obtained by SVD can form a simple linear superposition for original signal. The advantage of this linear superposition is that the isolation of one component signal from original signal corresponds to that this component is simply subtracted from original signal and this subtraction computation will make the isolated component signal keep its phase be the same as it is in original signal, namely that there is no phase shift in the isolated component signal. Eq. (10) is also the reconstruction formula of Hankel matrix-based SVD, and what its significance also consists in that some several interesting component signals can be simply added together to extract the feature information of original signal.

3. Similarity mechanism of signal decomposition between SVD and wavelet transform

As is well known, the essence of signal decomposition of wavelet transform is in fact to decompose original signal into the two spaces, i.e. scale space V_j and wavelet space W_j, in which the orthonormal bases of scale space V_j are composed of the dilation and translation set of scale function

$$\{\varphi_{j,k}(x) = 2^{-j/2}\varphi(2^{-j}x - k) | k \in Z\}$$

while the orthonormal bases of wavelet space W_j are composed of the dilation and translation set of wavelet function

$$\{\psi_{j,k}(x) = 2^{-j/2}\psi(2^{-j}x - k) | k \in Z\}$$

The approximation and detail signals obtained by wavelet transform are respectively the projections of original signal on these two orthonormal bases, and what they reflect are also the similarity extent between original signal and these two bases. The approximation and detail signals can be computed by following formula [13]

$$\begin{aligned} S_j f(x) &= 1/\sqrt{2}\sum_{n\in Z} h(n) S_{j-1} f(x - 2^{j-1}n) \\ W_j f(x) &= 1/\sqrt{2}\sum_{n\in Z} g(n) S_{j-1} f(x - 2^{j-1}n) \end{aligned} \tag{11}$$

where $h(n)$ and $g(n)$ are a couple of conjugate filters and satisfy $g(n) = (-1)^n \overline{h(1-n)}$, while original signal can be regarded as $S_0 f(n)$, so a series of approximation and detail signals can be obtained from Eq. (11) when $j > 0$.

From the angle of basis of vector space, much similarity of signal decomposition between the Hankel matrix-based SVD and wavelet transform can be found. For wavelet decomposition, its essence is to decompose a signal into the two spaces, i.e. scale and wavelet space, while the dilation and translation sets of scale and wavelet function, respectively, compose the orthonormal bases of these two spaces. The approximation and detail signals are obtained by dint of these two orthonormal bases and what they reflect, essentially, are the projections of original signal on these two orthonormal bases. While Hankel matrix is used in SVD, the essence of signal decomposition of SVD is also to decompose a signal into the two spaces, i.e. m-dimensional and n-dimensional space, in which the column vectors of the left orthogonal matrix $\boldsymbol{U}$ make up the orthonormal bases of m-dimensional space, and the column vectors of the right orthogonal matrix $\boldsymbol{V}$ make up the orthonormal bases of n-dimensional space. All the SVD component signals are obtained also by dint of these two orthonormal bases and what they reflect essentially are also the projections of original signal on these two orthonormal bases. So from the angle of vector space it can be seen that indeed there is much similarity between the Hankel matrix-based SVD and wavelet transform, namely that they both decompose a signal into the two spaces, and the decomposition results are both the projections of original signal on the orthonormal bases of these two spaces.

4. Orthogonality of component signals of two methods

It is easy to misconstrue that the decomposition results of SVD are orthogonal to one another because that $\boldsymbol{U}$ and $\boldsymbol{V}$ are both orthogonal matrix, but actually this is not the case, and the orthogonality of component signals of SVD is closely relative to the creation way of matrix $\boldsymbol{A}$. Next let us analyze the orthogonality of component signals obtained by Hankel matrix-based SVD.

4.1. Orthogonality of component signals obtained by Hankel matrix-based SVD

In order to study the orthogonality of component signals obtained by Hankel matrix-based SVD, firstly let us prove the following two lemmas.

Lemma 1. *Assume that $\boldsymbol{A}_i$ is expressed by row vectors $\boldsymbol{P}_{i,1}$, $\boldsymbol{P}_{i,2}$, …, $\boldsymbol{P}_{i,m}$, $\boldsymbol{P}_{i,m} \in \boldsymbol{R}^{1\times n}$, then arbitrary two row vectors in different matrix $\boldsymbol{A}_i$ and $\boldsymbol{A}_j$ are orthogonal, i.e.*

$$\boldsymbol{P}_{i,k}\boldsymbol{P}_{j,l}^T = 0, \quad i \neq j; \quad k = 1,2,\ldots,m; \quad l = 1,2,\ldots,m \tag{12}$$

Proof. Because that $\boldsymbol{v}_i$ are orthonormal to one another according to SVD theory, then

$$\boldsymbol{v}_i^T\boldsymbol{v}_j = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases}.$$

Thus

$$\boldsymbol{A}_i\boldsymbol{A}_j^T = \sigma_i\boldsymbol{u}_i\boldsymbol{v}_i^T\boldsymbol{v}_j\boldsymbol{u}_j^T\sigma_j = \begin{cases} \boldsymbol{u}_i\boldsymbol{u}_i^T\sigma_i^2 & i = j \\ \boldsymbol{O} & i \neq j \end{cases}. \tag{13}$$

For $\boldsymbol{A}_i$ can be expressed by row vectors $\boldsymbol{P}_{i,1}$, $\boldsymbol{P}_{i,2}$, …, $\boldsymbol{P}_{i,m}$, $\boldsymbol{P}_{i,m} \in \mathbf{R}^{1\times n}$, thus an arbitrary element of $\boldsymbol{A}_i\boldsymbol{A}_j^{\mathrm{T}}$ is $\boldsymbol{P}_{i,k}\,\boldsymbol{P}_{j,l}^{\mathrm{T}}$, where $k = 1, 2, \ldots, m$; $l = 1, 2, \ldots, m$. While according to Eq. (13), all elements of $\boldsymbol{A}_i\boldsymbol{A}_j^{\mathrm{T}}$ are 0 when $i \neq j$, then

$$\boldsymbol{P}_{i,k}\boldsymbol{P}_{j,l}^T = 0 \quad i \neq j; \quad k = 1,2,\ldots,m; \quad l = 1,2,\ldots,m$$

Lemma 2. *Assume that $\boldsymbol{A}_i$ is expressed by column vectors $\boldsymbol{L}_{i,1}$, $\boldsymbol{L}_{i,2}$,…,$\boldsymbol{L}_{i,n}$, $\boldsymbol{L}_{i,n} \in \boldsymbol{R}^{m\times 1}$, then arbitrary two column vectors in different matrix $\boldsymbol{A}_i$ and $\boldsymbol{A}_j$ are orthogonal, i.e.*

$$\boldsymbol{L}_{i,k}^T\boldsymbol{L}_{j,l} = 0 \quad i \neq j; \quad k = 1,2,\ldots,n; \quad l = 1,2,\ldots,n \tag{14}$$

Proof. Because that $\boldsymbol{u}_i$ are orthonormal to one another according to SVD theory, then

$$\boldsymbol{u}_i^T\boldsymbol{u}_j = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases}.$$

Thus

$$\boldsymbol{A}_i^T\boldsymbol{A}_j = \boldsymbol{v}_i\boldsymbol{u}_i^T\boldsymbol{u}_j\boldsymbol{v}_j^T\sigma_i\sigma_j = \begin{cases} \boldsymbol{v}_i\boldsymbol{v}_i^T\sigma_i^2 & i = j \\ \boldsymbol{O} & i \neq j \end{cases} \tag{15}$$

For $\boldsymbol{A}_i$ can be expressed by column vectors $\boldsymbol{L}_{i,1}$, $\boldsymbol{L}_{i,2}$, …, $\boldsymbol{L}_{i,n}$, $\boldsymbol{L}_{i,n} \in \mathbf{R}^{m\times 1}$, thus an arbitrary element of $\boldsymbol{A}_i^{\mathrm{T}}\boldsymbol{A}_j$ is $\boldsymbol{L}_{i,k}^{\mathrm{T}} \cdot \boldsymbol{L}_{j,l}$, where $k = 1, 2, \ldots, n$; $l = 1, 2, \ldots, n$. While according to Eq. (15), all elements of $\boldsymbol{A}_i^{\mathrm{T}}\boldsymbol{A}_j$ are 0 when $i \neq j$, then we have

$$\boldsymbol{L}_{i,k}^T\boldsymbol{L}_{j,l} = 0, \quad i \neq j; \quad k = 1,2,\ldots,n; \quad l = 1,2,\ldots,n$$

On the basis of these two lemmas, the orthogonality of component signals got by Hankel matrix-based SVD can be studied, and we have the following theorem:

Theorem 1. *The component signals got by Hankel matrix-based SVD are not orthogonal to one another, and the correlation of component $\boldsymbol{P}_i$ and $\boldsymbol{P}_j$ is determined by the product of elements in top right corner of $\boldsymbol{A}_i$ and $\boldsymbol{A}_j$, i.e.*

$$\boldsymbol{P}_i\boldsymbol{P}_j^T = -\boldsymbol{A}_i(1,n)\boldsymbol{A}_j(1,n) \tag{16}$$

where $\boldsymbol{A}_i(1,n)$ is the element locating in the first row and nth column of $\boldsymbol{A}_i$ and $\boldsymbol{A}_j(1,n)$ is the element locating in the first row and nth column of $\boldsymbol{A}_j$.

Proof. The kth column vector $\boldsymbol{L}_{i,k}$ of $\boldsymbol{A}_i$ can be expressed as $\boldsymbol{L}_{i,k} = \begin{pmatrix} \boldsymbol{A}_i(1,k) \\ \boldsymbol{H}_{i,k} \end{pmatrix}$, in which $\boldsymbol{A}_i(1,k)$is the first element of $\boldsymbol{L}_{i,k}$, and $\boldsymbol{H}_{i,k}$ is the column vector formed by all the other elements of $\boldsymbol{L}_{i,k}$, as is shown in Fig. 1, then the Eq. (14) can be written as

$$\boldsymbol{L}_{i,k}^T\,\boldsymbol{L}_{j,l} = \boldsymbol{A}_i(1,k)\boldsymbol{A}_j(1,l) + \boldsymbol{H}_{i,k}^T\boldsymbol{H}_{j,l} = 0.$$

Thus

$$\boldsymbol{H}_{i,k}^T \cdot \boldsymbol{H}_{j,l} = -\boldsymbol{A}_i(1,k)\boldsymbol{A}_j(1,l), \quad i \neq j; \quad k = 1,2,\ldots,n; \quad l = 1,2,\ldots,n$$

Then the inner product of $\boldsymbol{P}_i$ and $\boldsymbol{P}_j$ can be computed as follows:

$$\begin{aligned}\boldsymbol{P}_i\boldsymbol{P}_j^T &= (\boldsymbol{P}_{i,1}, \boldsymbol{H}_{i,n}^T)(\boldsymbol{P}_{j,1}, \boldsymbol{H}_{j,n}^T)^T \\ &= \boldsymbol{P}_{i,1}\boldsymbol{P}_{j,1}^T + \boldsymbol{H}_{i,n}^T\boldsymbol{H}_{j,n} \\ &= -\boldsymbol{A}_i(1,n)\boldsymbol{A}_j(1,n)\end{aligned}$$

in which $\boldsymbol{P}_{i,k}\boldsymbol{P}_{j,l}{}^{\mathrm{T}} = 0$, the conclusion of Lemma 1, is cited. So $\boldsymbol{P}_i$ and $\boldsymbol{P}_j$ are not orthogonal and their correlation is determined by the product of elements in top right corner of $\boldsymbol{A}_i$ and $\boldsymbol{A}_j$.

4.2. Orthogonality of decomposition results of wavelet transform

As a comparison, the orthogonality of decomposition results of wavelet transform should also be studied, but it is difficult to study this kind of orthogonality by wavelet's fast computation Eq. (11), while cross-correlation function is a much more convenient tool to study this kind of orthogonality [14]. Assume that $Wf(s_1, x)$ and $Wf(s_2, x)$ are the decomposition results of signal $f(t)$ obtained by wavelet transform in two different scales, and their cross-correlation function can be written as

$$R_{s_1,s_2}(\tau) = \int_{-\infty}^{+\infty} Wf(s_1,x)Wf(s_2,x+\tau)\,\mathrm{d}x. \tag{17}$$

For orthogonality of wavelet decomposition results in different scales, we have the following theorem:

Theorem 2. *If wavelet $\psi(t)$ is the orthonormal basis of $L^2(R)$ space, then the decomposition results of this wavelet in two different scales are orthogonal each other; while auto correlation of the decomposition result in each scale is the same as that of original signal.*

Proof. According to the definition of wavelet transform and cross-correlation function, we have

$$\begin{aligned}R_{s_1,s_2}(\tau) &= \int_{-\infty}^{+\infty} \left\{ \left[\frac{1}{s_1} \int_R f(u)\psi\left(\frac{x-u}{s_1}\right) \mathrm{d}u \right] \left[\frac{1}{s_2} \int_R f(v)\psi\left(\frac{x+\tau-v}{s_2}\right) \mathrm{d}v \right] \right\} \mathrm{d}x \\ &= \int_{-\infty}^{+\infty} \int_R f(u)f(v) \left[\frac{1}{s_1 s_2} \int_R \psi\left(\frac{x-u}{s_1}\right)\psi\left(\frac{x+\tau-v}{s_2}\right) \mathrm{d}x \right] \mathrm{d}u\,\mathrm{d}v\end{aligned}$$

Let $w = v-\tau$, then

$$= \int_{-\infty}^{+\infty} \int_R f(u)f(w+\tau) \left[\frac{1}{s_1 s_2} \int_R \psi\left(\frac{x-u}{s_1}\right)\psi\left(\frac{x-w}{s_2}\right) \mathrm{d}x \right] \mathrm{d}u\,\mathrm{d}w$$

For $\psi(t)$ is the orthonormal basis of $L^2(R)$, thus

$$\begin{aligned}&= \int_{-\infty}^{+\infty} \int_R f(u)f(w+\tau)\delta(s_1,s_2)\delta(u,w)\,\mathrm{d}u\,\mathrm{d}w \\ &= \delta(s_1,s_2) \int_R f(u)f(u+\tau)\,\mathrm{d}u \\ &= \delta(s_1,s_2)R_f(\tau)\end{aligned}$$

where $\delta(x,y) = \begin{cases} 1 & x = y \\ 0 & x \neq y \end{cases}$.

There are large numbers of orthonormal wavelets, such as Daubechies wavelet, Meyer wavelet, spline wavelet, Haar wavelet, and etc, and their decomposition results in different scales are orthogonal to one another. While there are also many non-orthogonal wavelets such as Mexican hat wavelet, Gauss wavelet, Morlet wavelet, and etc., and their decomposition results in different scales are correlated to one another.

5. Similarity comparison of signal processing effect between the two methods

After the theoretical analysis, now the practical effect of signal processing of these two methods may be compared with each other. A simulated second-order unbalanced vibration signal of rotor with 512 data is shown in Fig. 4, and white noises are added in this signal. A Hankel matrix with column $n = 5$ and row $m = 508$ is created by this signal, then 5 component signals can be obtained by the SVD method, which are illustrated in Fig. 5(a). While the decomposition results of this signal obtained by wavelet transform in 4 scales are illustrated in Fig. 5(b). One can easily see that the processing

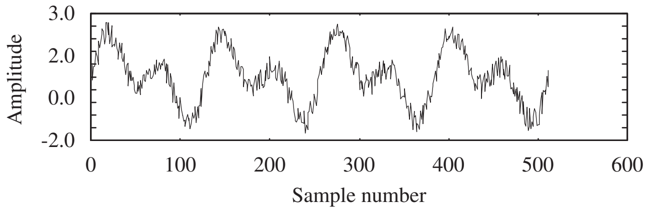

Fig. 4. A simulated second-order unbalanced vibration signal.

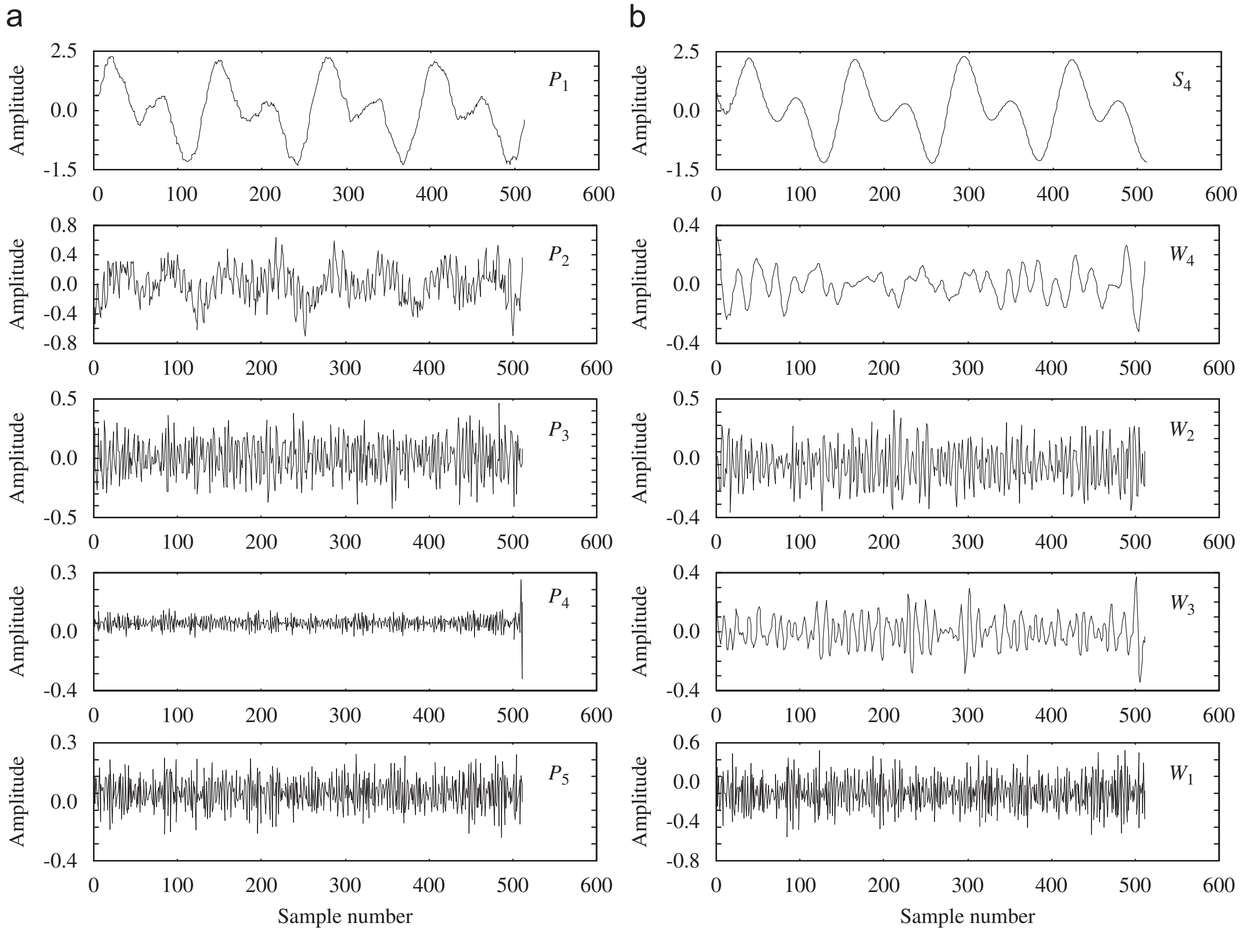

Fig. 5. Similarity between the decomposition results of SVD and wavelet transform. (a) Component signals got by SVD and (b) component signals got by wavelet transform.

effect of these two methods is really similar to each other, in which the first SVD component signal $\boldsymbol{P}_1$ is very similar to $\boldsymbol{S}_4$, which is the approximation signal in the last scale got by wavelet transform, and in these two signals, noises are basically eliminated and what they reflect are both the skeleton of original signal. While the other 4 SVD component signals are similar to the detail signals of wavelet transform, though they are not corresponding in-order, what they reflect are similar and are all high frequency noises in original signal.

Besides the preceding theoretical analysis for this kind of similarity, some further explanation can be made according to the structure characteristic of Hankel matrix in itself. In Hankel matrix, the next row is formed by right shift of the last row only one data point, so these row vectors are highly correlated to each other. The characteristic of singular values of this kind of matrix whose row vectors are highly correlated is that, the first singular value is much bigger than the others and although there is difference in other singular values, but this difference is very little. The 5 singular values in this example are illustrated in Fig. 6, in which the first singular value is 6.8 times of the second one, and starting from the second singular value, there is no much difference among these singular values. While according to Eq. (3), the expression formula of component signal $\boldsymbol{P}_i$, and Lemma 1,2, it is easy to deduce that the energy of component signal is proportional to square of corresponding singular value, i.e. $|\boldsymbol{P}_i|^2 \propto \sigma_i{}^2$, then the energy proportion of the ith component signal $\boldsymbol{P}_i$ in the original signal can be expressed as

$$\eta_i = \frac{\sigma_i^2}{\sigma_1^2 + \sigma_2^2 + \cdots + \sigma_q^2}, \quad i = 1, 2, \ldots, q \tag{18}$$

So for the structure characteristic of Hankel matrix in itself, the energy of original signal will be mainly concentrated on the first component signals $\boldsymbol{P}_1$, which corresponds to the first singular value, therefore, $\boldsymbol{P}_1$ can represent the main skeleton of original signal and can achieve the similar effect as that of the approximation signal in wavelet transform. While those detail features with tiny energy in original signal will be isolated to the other component signals, which correspond to those tiny singular values, therefore, these component signals can achieve the similar effect as that of detail signals in wavelet transform. However, it can be known from preceding theoretical analysis that there is no phase shift in SVD component signals, and to reveal the phase difference between these two methods, $\boldsymbol{P}_1$ and $\boldsymbol{S}_4$ are plotted in the same figure, as is shown in Fig. 7, and it can be clearly seen that there is no phase shift in $\boldsymbol{P}_1$, while an obvious phase lag is existed in $\boldsymbol{S}_4$, so from the viewpoint of phase shift, the processing results of SVD have the advantage of those of wavelet transform.

In milling process, the wear of milling cutter will result in the shocks in the milling force, but these shocks are always submerged by noises and it is very difficult to identify them from a raw milling force signal, here Hankel matrix-based SVD and wavelet transform method are used to extract these shocks. An end milling cutter with four edges whose wear is moderate is used to mill tempered 45# steel, simultaneously the milling force is detected by KISTLER piezoelectricity crystal force measuring apparatus and one milling force signal with 2048 data is shown in Fig. 8. The characteristic of this signal is: there are 4 small waves in each cycle, which are caused by the four edges of end milling cutter, while the shocks caused by tool wear can hardly be seen. A Hankel matrix with column $n = 5$ and row $m = 2044$ is created by this signal and after the processing of SVD, 5 component signals can be obtained, which are illustrated in Fig. 9(a). It can be seen that many

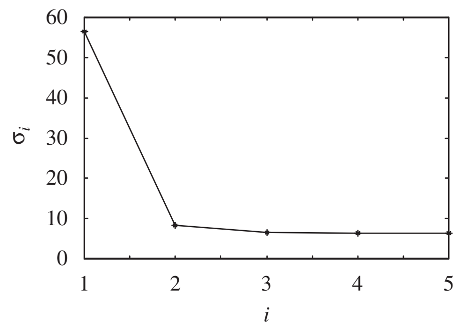

Fig. 6. Distribution of singular values.

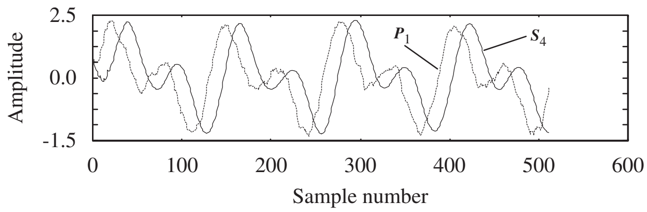

Fig. 7. Phase shift contrast between two methods.

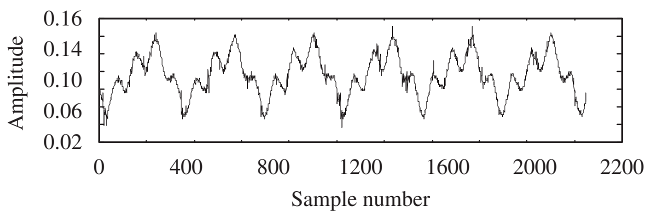

Fig. 8. A milling force signal.

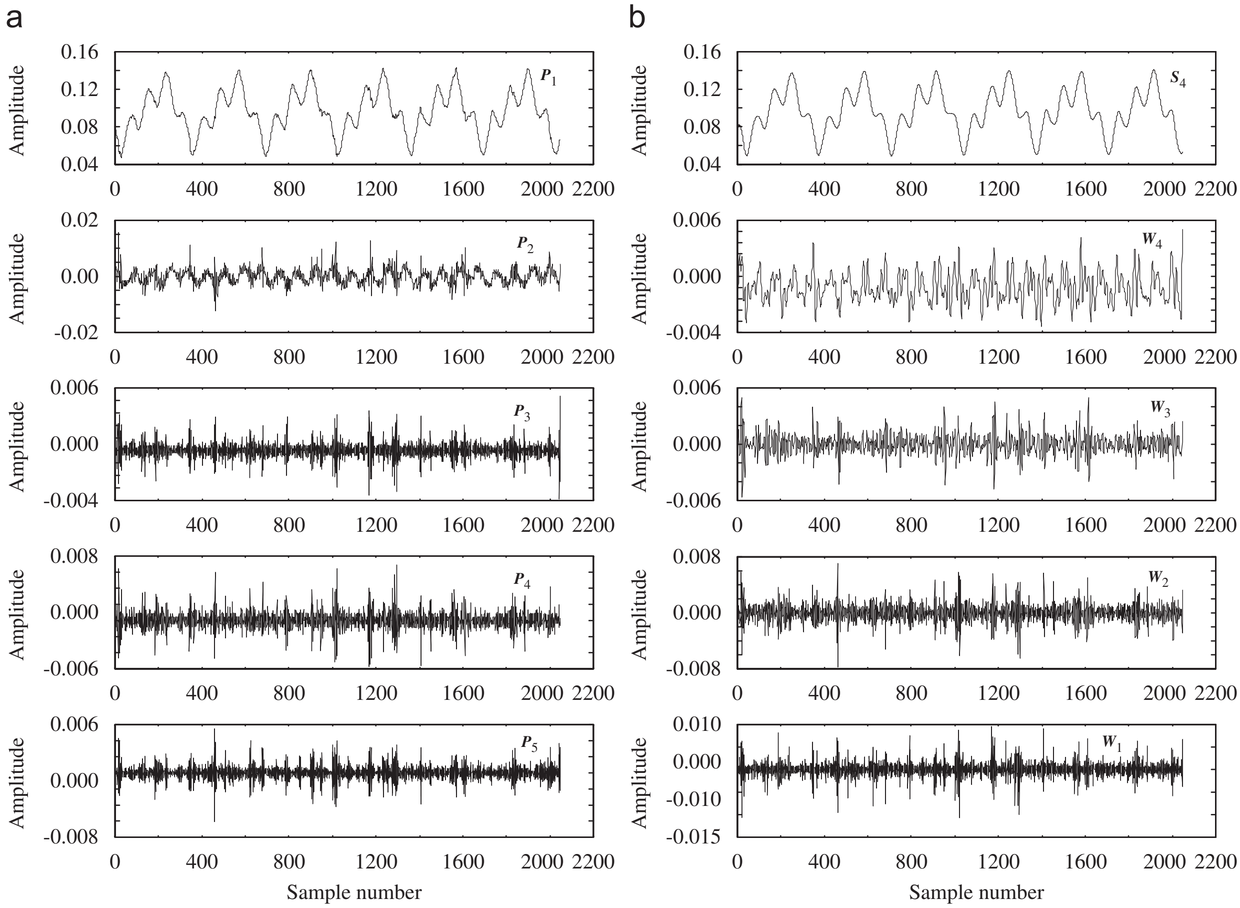

Fig. 9. Similarity between the processing results of SVD and wavelet transform for milling force signal. (a) Component signals got by SVD and (b) component signals got by wavelet transform.

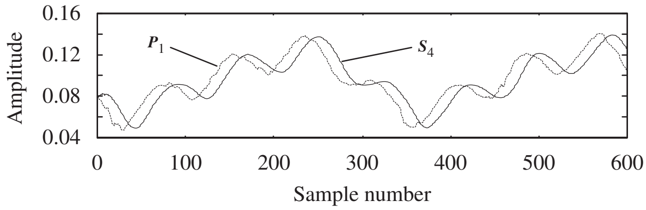

Fig. 10. The phase shift contrast between $\boldsymbol{P}_1$ and $\boldsymbol{S}_4$.

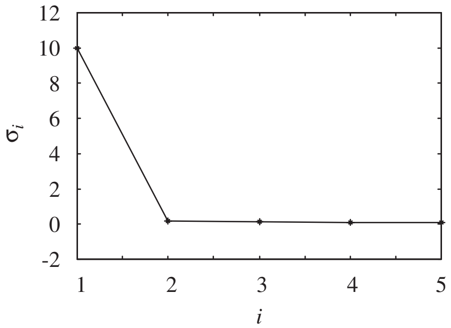

Fig. 11. The 5 singular values of milling force signal.

impulses have been isolated in component signal $\boldsymbol{P}_3$, $\boldsymbol{P}_4$ and $\boldsymbol{P}_5$, and these impulses can never be noises for their positions in $\boldsymbol{P}_3$, $\boldsymbol{P}_4$ and $\boldsymbol{P}_5$ are so regular corresponding, while noises are random and can never have so regular position corresponding relationship in different component signals. The real conditions of milling process being combined with, it can be inferred that these impulses are nothing but the shocks resulted from the wear of milling cutter, in original signal they are so tiny that they cannot be recognized, but after SVD processing they are extracted. As a comparison, the wavelet decomposition results of original milling force signal are shown in Fig. 9(b), one can easily see that they are very similar to those of SVD, and what detail signals $\boldsymbol{W}_3$, $\boldsymbol{W}_2$ and $\boldsymbol{W}_1$ reflect are the same shocks feature as those extracted by SVD component $\boldsymbol{P}_3$, $\boldsymbol{P}_4$ and $\boldsymbol{P}_5$; while the approximation signal $\boldsymbol{S}_4$ is the skeleton of milling force signal, so is the SVD component signal $\boldsymbol{P}_1$, but the phase shift of $\boldsymbol{P}_1$ is zero while there is phase lag in $\boldsymbol{S}_4$, the front 600 data of $\boldsymbol{P}_1$ and $\boldsymbol{S}_4$ are illustrated in Fig. 10, in which the phase difference of these two signals are clearly shown. The 5 singular values of the milling force signal are illustrated in Fig. 11, in which the first singular value is 55.5726 times of the second one.

6. Comparison of singularity detection between the two methods

Besides the very similar effect on signal decomposition, another surprising similarity of these two methods is: it is discovered that singularity can also be detected by Hankel matrix-based SVD. Here 'singularity' is another concept that is completely different from the 'singular' in SVD. For signal $f(t)$, if there is a sudden change in $f(t)$ or its certain-order derivatives at certain time 't', then this $f(t)$ is called to have singularity at time 't'. Lipschitz index (called Lip index for short) can be used to describe the singular degree of $f(t)$, supposing that there is a non-negative integer n, and $n \leqslant \alpha < n+1$, if there exist a constant $A>0$ and a polynomial $p_n(t)$ with degree n, for $|t-t_0|<\delta$, where δ is a very tiny number, $f(t)$ satisfies following inequality

$$|f(t) - p_n(t - t_0)| \leqslant A|t - t_0|^{\alpha} \tag{19}$$

Then $f(t)$ is called Lipα at time 't_0', if Lip index $\alpha<1$, then t_0 is the singular point of $f(t)$, while if $n \leqslant \alpha < n+1$ and $n>1$, then t_0 is the singular point of the nth derivative of $f(t)$. Singular points of signal generally contain much important information, such as in fault diagnosis domain, singular points of signal generally imply the impact, oscillation, sudden change of rotating speed, deformation or fracture of structure and so on caused by fault [15], so the detection for singular point in signal is very significant.

Originally, the detection for singularity is an outstanding advantage of wavelet transform, and vanishing moment is an important parameter to appraise the singularity detection performance of wavelet. For a wavelet function $\psi(t)$, if it satisfies the following equation:

$$\int_R t^k \psi(t)\,\mathrm{d}t = 0, \quad k = 0, 1, \ldots, n-1 \tag{20}$$

then it is said that the order of vanishing moment of this wavelet is 'n'. The order of vanishing moment of each kind of wavelet is fixed, and a wavelet with vanishing moment of order n can detect the singularity of the $n-1$th derivative of $f(t)$. The singularity detection of wavelet has been widely applied to the domains of fault diagnosis and image edge detection. To our surprise, it is discovered that singularity can also be detected by SVD when Hankel matrix is used, and furthermore, compared with wavelet transform, SVD has special advantages in singularity detection. For a given wavelet, its vanishing moment order is fixed and then the detected Lip index is also fixed. For example, a wavelet with vanishing moment of order 1 can only detect the singularity of signal in itself but can do nothing for the detection of singularity of derivative of signal. While for SVD, there is a great difference from wavelet in singularity detection, it is discovered that starting from the second SVD component signal, singularity can be detected in each SVD component signal, and furthermore the detected Lip index are not fixed and will be increased successively. If the concept of vanishing moment is applied to SVD components, then we can say that the order of vanishing moment of the second SVD component signal is 1, and the one of the third SVD component signal is 2,..., and the one of the nth SVD component signal is '$n-1$', so singular points with different Lip index can also be detected by the SVD component signals.

An example is given to test the above statements, and simultaneously the performances of SVD and wavelet transform in singularity detection are also compared with each other. Supposing that there is a signal as follows:

$$f(t) = \begin{cases} 2 & 0<t<50 \\ 7 & 50 \leqslant t \leqslant 99 \\ -2t+205 & 99 \leqslant t \leqslant 150 \end{cases} \tag{21}$$

This signal is illustrated in Fig. 12, in this signal there is a sudden change at $t = 50$, and Lip index of this point is $\alpha = 0$, while at $t = 99$ this signal is continuous in itself but its first derivative is not continuous at this point, and Lip index of this point is $\alpha = 1$, these two points are both singular points.

A Hankel matrix with column $n = 4$ and row $m = 147$ is created by this signal, and 4 component signals can be got after the processing of SVD, which are shown in Fig. 13. It can be seen that what the first component signal $\boldsymbol{P}_1$ reflects is the skeleton of the original signal, while starting from the second component signal; the singular points of original signal have been detected in each component signal. In the second component signal $\boldsymbol{P}_2$, a peak is produced at $t = 50$ to indicate the singularity of this point, but the singularity at $t = 99$ has not been detected, and this detection result is just as same as that

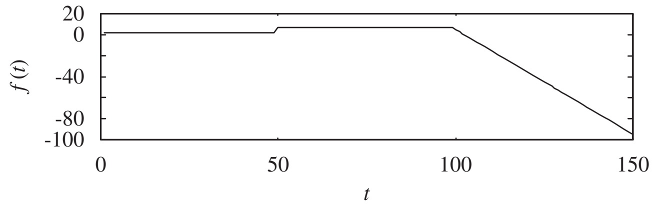

Fig. 12. Original singular signal.

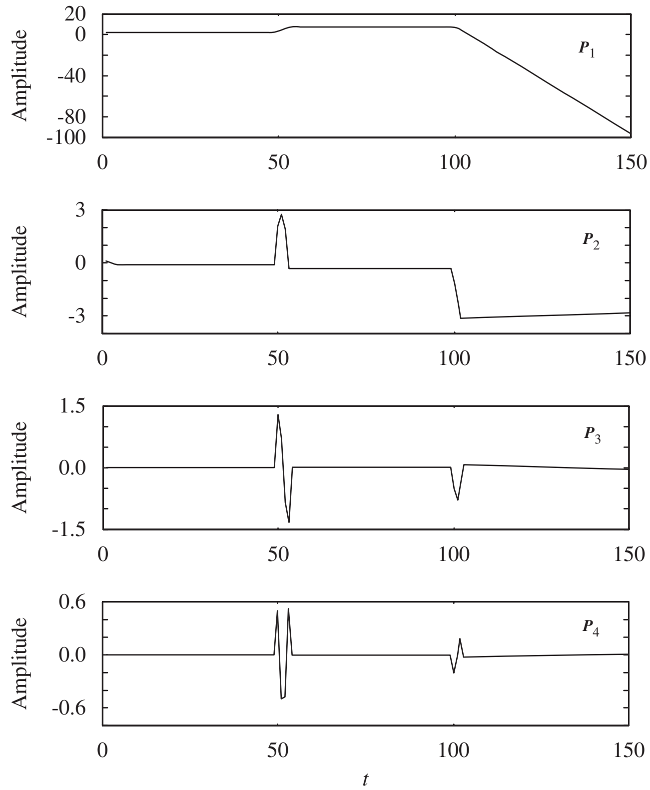

Fig. 13. Singularity detection results achieved by Hankel matrix-based SVD.

of wavelet with vanishing moment of order 1. While in the third component signal $\boldsymbol{P}_3$ one zero-crossing is produced at $t = 50$, this is just the detecting characteristic of wavelet with vanishing moment of order 2 for sudden change signal, while at $t = 99$ a negative peak is produced to show that there is a negative sudden change in the derivative of original signal, and this is just as same as the detecting characteristic of wavelet with vanishing moment of order 2 for singular point with Lip index $\alpha = 1$. In the fourth component signal $\boldsymbol{P}_4$ two zero-crossings are produced at $t = 50$, this is also the detecting characteristic of wavelet with vanishing moment of order 3 for sudden change signal, while one zero-crossing is produced at $t = 99$, this is also the detecting characteristic of wavelet with vanishing moment of order 3 for singular point with Lip index $\alpha = 1$. So according to the above analysis, two facts can be revealed by all these detection results: (1) it is demonstrated that singularity of signal can be detected by Hankel matrix-based SVD. (2) Starting from the second component signal, the order of vanishing moment of each SVD component is increased successively.

As a comparison, the detection results achieved by spline wavelet, whose vanishing moment order is 1, are shown in Fig. 14. One can easily see that the singularity detection effect of detail signals $\boldsymbol{W}_1$–$\boldsymbol{W}_4$ are just as same as that of the second SVD component $\boldsymbol{P}_2$, and because of the limit of order of vanishing moment, the singularity at $t = 99$ has not been detected all along. This contrast also reveals that the order of vanishing moment of the second SVD component $\boldsymbol{P}_2$ is 1.

While the detection results achieved by Mexican hat wavelet, whose vanishing moment order is 2, are illustrated in Fig. 15, it can be seen that one zero-crossing is produced at $t = 50$ and one negative peak is produced at $t = 99$ and this kind of detection effect is just as same as that of the third SVD component $\boldsymbol{P}_3$. This contrast also reveals that the order of vanishing moment of the third SVD component $\boldsymbol{P}_3$ is 2.

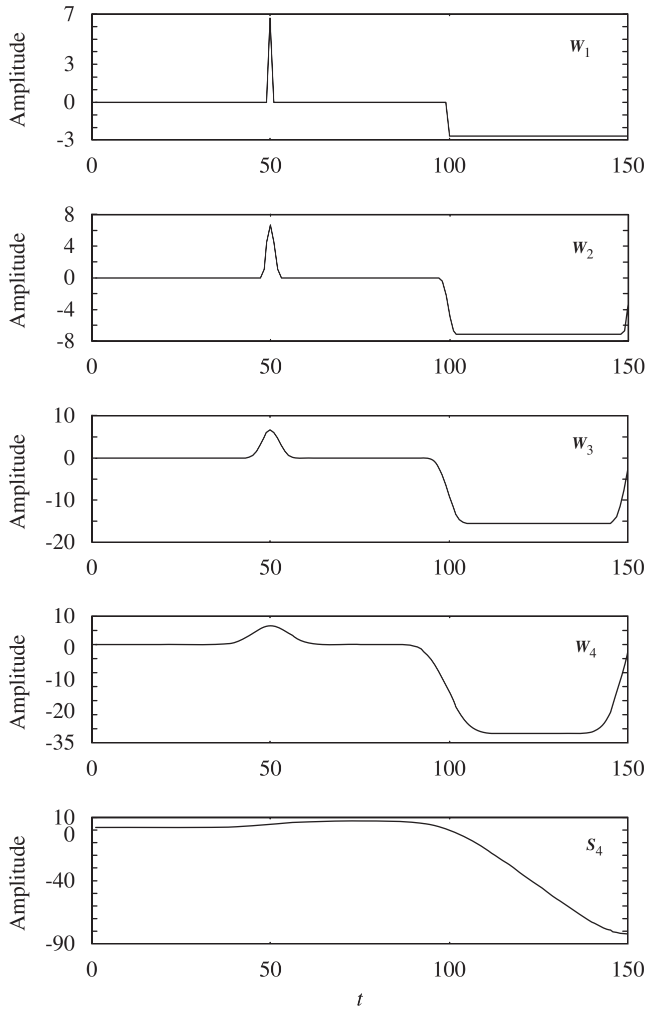

Fig. 14. Singularity detection results achieved by spline wavelet.

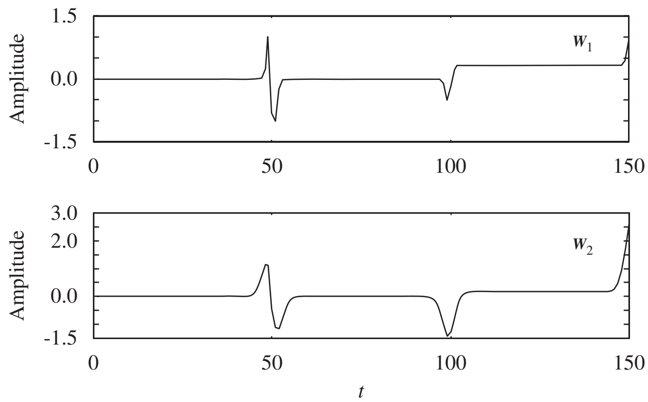

Fig. 15. Singularity detection results achieved by Mexican hat wavelet.

The detection results achieved by No. 3 Daubechies wavelet, whose vanishing moment order is 3, are illustrated in Fig. 16, it can be seen that the detection effect is just as same as that of the fourth SVD component $\boldsymbol{P}_4$, i.e. two zero-crossings are produced at $t = 50$ and one zero-crossing is produced at $t = 99$. This contrast also reveals that the order of vanishing moment of the fourth SVD component $\boldsymbol{P}_4$ is 3.

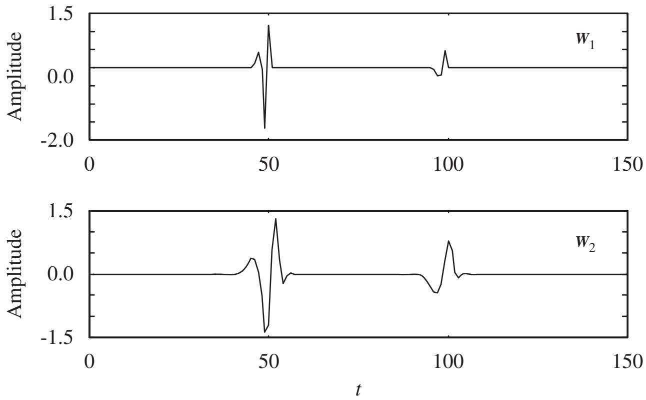

Fig. 16. Singularity detection results achieved by No. 3 Daubechies wavelet.

Continuously compared like this way with different Daubechies wavelets, it can be found that detection effect of No. 'n' Daubechies wavelet is as same as that of the n+1th SVD component, and this means that the order of vanishing moment of both No. 'n' Daubechies wavelet and the n+1th SVD component are the same, because that the order of vanishing moment of No. 'n' Daubechies wavelet is 'n', so it can be deduced that the order of vanishing moment of the nth SVD component is '$n-1$'. Besides, it can also be found from Figs. 13, 15 and 16 that in SVD component signals the symmetry of impulse indicating the position of singular point is better than that in detection results of Mexican hat and Daubechies wavelet. Furthermore, because data extension are necessary in wavelet decomposition, so boundary error will surely be existed in wavelet decomposition results, in this example the right boundary error in detail signals $\boldsymbol{W}_3$, $\boldsymbol{W}_4$ in Fig. 14 and $\boldsymbol{W}_2$ in Fig. 15 is especially obvious, while for SVD, data extension is not needed so that the boundary error will never happen in its decomposition results, so in this aspect, SVD also has the advantage of wavelet transform.

Besides the progressive increase of the order of vanishing moment, there is another characteristic in SVD for singularity detection, namely that the widths of impulses indicating the position of singularity are the same in all component signals. Such as in this example, the widths of impulses indicating the position of singularity are 4 sampling intervals throughout all SVD component signals. While the widths of impulses in wavelet detection results are progressively increased with the scale, such as in the detection results of spline wavelet in this example, the width of impulse indicating the position of singularity in the first scale is 2 sampling intervals, 6 sampling intervals in the second scale, 14 sampling intervals in the third scale, and even 28 sampling intervals are reached in the fourth scale.

This phenomenon is by no means fortuitous, we use Hankel matrices with different column number and different singular signals to test again and again, and the following fact is discovered: in all SVD component signals, the widths of impulses indicating the position of singularity will always keep the same all along, and this width is determined by the column number of Hankel matrix, if column number of Hankel matrix is n, then the width of impulse is nT_s, where T_s is the sampling period of original signal.

7. Conclusions

SVD and wavelet transform are two completely different theories, but their signal processing effect is surprisingly similar when Hankel matrix is used in SVD. The signal decomposition principle of Hankel matrix-based SVD and the essence of component signals obtained by this method being studied, the mechanism of similarity between SVD and wavelet transform is analyzed from the angle of basis of vector space and characteristic of Hankle matrix. By theoretical analysis and signal processing examples, following conclusions can be drawn:

1. A signal can be decomposed into the linear sum of a series of component signals by Hankel matrix-based SVD, and what these component signals reflect in essence are the projections of original signal on the orthonormal bases of m-dimensional and n-dimensional spaces. This characteristic is just very similar to that what wavelet decomposition results reflect are projections of original signal on the orthonormal bases of scale and wavelet spaces.
2. The component signals obtained by Hankel matrix-based SVD are not orthogonal to one another, while the ones obtained by orthogonal wavelet are orthogonal to one another, but the ones obtained by non-orthogonal wavelet are also not orthogonal.
3. For component signals obtained by Hankel matrix-based SVD, the first component signal corresponds to the approximation signal of last scale obtained by the wavelet transform, and what they reflect are both the skeleton of original signal; while the other component signals correspond to the detail signals obtained by wavelet transform, and what they reflect are all the detail features of original signal, but there is no phase shift in SVD component signals, while phase lag is existed in wavelet decomposition results.

4. When Hankel matrix is used, singularity of signal can also be detected by SVD. Compared with wavelet transform, there are two characteristics in SVD for singularity detection, one is that the order of vanishing moment of SVD component signals is progressively increased and the one of the nth SVD component signal is '$n-1$', so singular points with different Lip index can be detected by different SVD components, the other is that the width of impulse indicating the position of singular point will always keep the same throughout all component signals and this width is determined by the column number of Hankel matrix. While for wavelet, the detected Lip index is fixed, and moreover, the width of impulse indicating the position of singular point will become broad and broad with the increase of scale.
5. The similarity of Hankel matrix-based SVD and wavelet transform in signal processing can be applied to domains of noise reduction, singularity detection, feature extraction and fault diagnosis, and this similarity can also be used to test and validate each other's signal processing effect on these domains.

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References

[1] G.H. Golub, C.F. Van Loan, Matrix Computations, third ed., John Hopkins University Press, Baltimore, Maryland, USA, 1996.
[2] X.Z. Zhao, T.J. Chen, B.Y. Ye, Processing of milling force signal and isolation of state information of milling machine based on singular value decomposition, Chinese Journal of Mechanical Engineering 43 (6) (2007) 169–174 (in Chinese).
[3] C. Cempel, Multidimensional condition monitoring of mechanical system in operation, Mechanical Systems and Signal Processing 17 (6) (2003) 1291–1303.
[4] M.J. Brenner, Non-stationary dynamics data analysis with wavelet-SVD filtering, Mechanical Systems and Signal Processing 17 (4) (2003) 765–786.
[5] W.S. Wang, F.B. Chen, J.Y. Yang, A method of feature extraction based on SVD, Journal of Electronics and Information Technology 27 (2) (2005) 294–297 (in Chinese).
[6] H.X. Liu, J. Li, Y. Zhao, L.S. Qu, Improved singular value decomposition technique for detecting and extracting periodic impulse component in a vibration signal, Chinese Journal of Mechanical Engineering (English Edition) 17 (3) (2004) 340–345.
[7] J.J. Wei, C.J. Chang, N.K. Chou, ECG data compression using truncated singular value decomposition [J], IEEE Transactions on Information Technology in Biomedicine 5 (4) (2001) 290–299.
[8] K. Shin, J.K. Hammond, P.R. White, Iterative SVD method for noise reduction of low-dimensional chaotic time series, Mechanical Systems and Signal Processing 13 (1) (1999) 115–124.
[9] K. Shin, S.A. Feraday, C.J. Harris, M.J. Brennan, Optimal autoregressive modelling of a measured noisy deterministic signal using singular-value decomposition, Mechanical Systems and Signal Processing 17 (4) (2003) 423–432.
[10] K.Y. Sanliturk, O. Cakar, Noise elimination from measured frequency response functions, Mechanical Systems and Signal Processing 19 (2005) 615–631.
[11] Q.B. Zhu, J. Liu, Y.G. Li, B.C. Wen, Study on noise reduction in singular value decomposition based on structural risk minimization, Journal of Vibration Engineering 18 (2) (2005) 204–207 (in Chinese).
[12] G.P. Wang, C.C. Bao, P. Zhang, Low bit rates waveform interpolation speech coding based on singular value decomposition, Acta Electronica Sinica 34 (1) (2006) 135–140 (in Chinese).
[13] X.Z. Zhao, T.J. Chen, B.Y. Ye, Research on signal processing effect of both inner product type of wavelet transform and convolution type of wavelet transform, Chinese Journal of Mechanical Engineering 40 (3) (2004) 55–60 (in Chinese).
[14] N. Wang, Z.H. Sheng, Identification of orbits of shaft centerline based on wavelet vectors and AR model, Signal Processing 15 (2) (1999) 157–162 (in Chinese).
[15] X.Z. Zhao, Y. Lin, W.G. Chen, The choice of wavelet bases for singularity detection, Journal of South China University of Technology 28 (10) (2000) 75–80 (in Chinese).

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